$\mathfrak{g} = \widehat{\mathfrak{sl}}(n)$, $U_q(\mathfrak{g})$-module $V(\Lambda_i)$, $0 \le i \le n-1$.

$(L(\Lambda_i), B(\Lambda_i))$ crystal base for $V(\Lambda_i)$

$B(\Lambda_i)$ = set of n-reduced extended Young diagrams of charge i.

Let $\lambda = \sum_{i=0}^{n-1} m_i \Lambda_i \in P^+$, $m_i \in \mathbb{Z}_{\geq 0}$.

Consider the $U_q(\widehat{\mathfrak{sl}}(n))$-module $V(\lambda)$.

Let $(L(\lambda), B(\lambda))$ be the crystal base for $V(\lambda)$.

$$V(\lambda) \subseteq V(\Lambda_0)^{\otimes m_0} \otimes V(\Lambda_1)^{\otimes m_1} \otimes \cdots \otimes V(\Lambda_{n-1})^{\otimes m_{n-1}}$$

$$u_\lambda = \underbrace{u_{\Lambda_0} \otimes \cdots \otimes u_{\Lambda_0}}_{m_0} \otimes \underbrace{u_{\Lambda_1} \otimes \cdots \otimes u_{\Lambda_1}}_{m_1} \otimes \cdots \otimes \underbrace{u_{\Lambda_{n-1}} \otimes \cdots \otimes u_{\Lambda_{n-1}}}_{m_{n-1}}$$

$$\Rightarrow B(\lambda) \subseteq B(\Lambda_0)^{\otimes m_0} \otimes \cdots \otimes B(\Lambda_{n-1})^{\otimes m_{n-1}}$$

$$\lambda = \Lambda_{\gamma_1} + \Lambda_{\gamma_2} + \cdots + \Lambda_{\gamma_\ell} \in P^+$$

where $0 \leq \gamma_1 \leq \gamma_2 \leq \cdots \leq \gamma_\ell$

$(\ell = m_0 + m_1 + \cdots + m_{n-1})$

$$\mathcal{Y}(\lambda) = \{ Y = (Y_1, Y_2, \cdots, Y_\ell) \mid Y_j \text{ has charge } \gamma_j \}$$

$$\mathrm{wt}(Y) = \sum_{j=1}^{\ell} \mathrm{wt}(Y_j)$$

We denote the ith corner with diagonal number d in Y_j by the pair (d, j).

Two i-corners in Y (d, j) and (d', j') are ordered as follows:

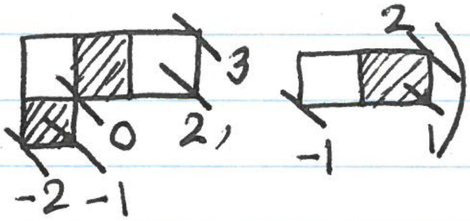
$$(d, j) \geq (d', j') \text{ if } d > d' \text{ or } d = d' \,\&\, j \leq j'.$$

which is a total order.

<u>Example:</u> $Y = (Y_1, Y_2)$, $n = 2$

$Y_1 = (-2, -1, -1, 0, 0, \cdots, 0, \cdots)$

$Y_2 = (-1, -1, 0, 0, 0, \cdots\cdots)$

$Y = \Big($  $\Big) = (Y_1, Y_2)$

0-corners : $(2,2)$,
$(-2,1), (0,1), (2,1)$

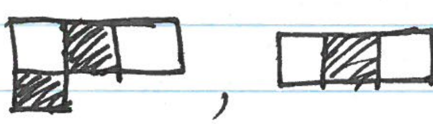
$$(2,1) > (2,2) > (0,1) > (-2,1)$$

0-signature : $- + + +$ (reduced)

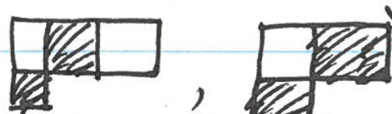
1-corners : $(1,2), (-1,2), (3,1), (-1,1)$

$$(3,1) > (1,2) > (-1,1) > (-1,2)$$

1-signature : $+ - - +$
$= - +$ (reduced)

$\tilde{f}_0 Y = \Big($  $\Big)$

$\tilde{e}_0 Y = \Big($  $\Big)$

$\tilde{f}_1 Y = \Big($  $\Big)$

$\tilde{e}_1 Y = \Big($  $\Big)$

<u>Thm</u>: The crystal $B(\lambda)$ of the $U_q(\hat{sl}(n))$-module $V(\lambda)$ for $\lambda = \Lambda_{\gamma_1} + \Lambda_{\gamma_2} + \cdots + \Lambda_{\gamma_\ell}$, $0 \le \gamma_1 \le \gamma_2 \le \cdots \le \gamma_\ell \le n-1$ is realized as:

$$B(\lambda) = \{ Y = (Y_1, Y_2, \cdots, Y_\ell) \in \mathcal{Y}(\lambda) \mid Y_1 \supseteq Y_2 \supseteq \cdots \supseteq Y_\ell \supseteq Y_1[n] \text{ and for each } k \geq 0, \exists\, 1 \le j \le \ell \text{ such that } (Y_{j+1})_k > (Y_j)_{k+1} \}$$

<u>Remark</u>: In $B(\lambda)$, each Y_j is forced to be n-reduced by the conditions.

<u>Example</u>: $U_q(\hat{sl}(2))$-module $V(\lambda)$ for $\lambda = \Lambda_0 + \Lambda_1$.

Here $c = h_0 + h_1$ is canonical central element of $\hat{sl}(2)$. Hence level of λ is $(\Lambda_0 + \Lambda_1)(c) = 1 + 1 = 2$.

Partial crystal graph $B(\lambda)$: