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$$Y = (y_k)_{k \geq 0}, \quad Y' = (y'_k)_{k \geq 0}$$

$Y \supseteq Y'$ if $y_k \leq y'_k \quad \forall\, k \geq 0$.

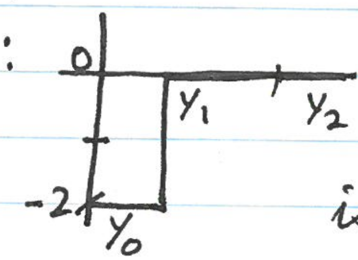
$$Y[j] = (y_k + j)_{k \geq 0}, \quad j \geq 0.$$

$Y \supseteq Y[j]$.

Defn: $Y = (y_k)_{k \geq 0}$ is n-reduced if

$$y_{k+1} < y_k + n \quad \forall\, k \geq 0.$$

Ex:



$Y = (-2, 0, 0, \ldots)$ is j-reduced charge 0 extended Young diagram for $j \geq 3$. Note Y is not 2-reduced.

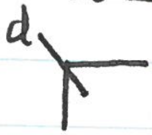
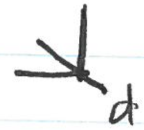
$Y = (y_k)_{k \geq}$ Young diagram of charge $0 \leq \gamma \leq n-1$.

$Y = (Y_\gamma, \gamma)$, Y_γ is the finite Young diagram corresponding to Y.

$m_i = \#$ of boxes of color $0 \leq i \leq n-1$ in Y_γ

Then $\quad wt(Y) = \Lambda_\gamma - \sum_{i=0}^{n-1} m_i \alpha_i$

For any $k \geq 0$, if $y_{k+1} \neq y_k$, then we have a corner like  or  with diagonal number d.

If $d \equiv i \bmod n$, $0 \leq i \leq n-1$, then we say  is a concave i-corner and  is a convex i-corner.

Consider the $U_q(\widehat{sl}(n))$-module $V(\Lambda_\gamma)$ and $(L(\Lambda_\gamma), B(\Lambda_\gamma))$ be its crystal base.

<u>Thm</u>: $B(\Lambda_\gamma) = \{ Y = (y_k)_{k \geq 0} \mid y_\infty = \gamma,\ y_{k+1} \leq y_k + n \}$ $\forall\, k \geq 0$

Let $Y = (y_k)_{k \geq 0}$, fix $0 \leq i \leq n-1$.

- collect the i-corners in Y
- order the i-corners in the decreasing order of its diagonal numbers (i.e. order them from right to left).
- assign i-signature '+' to a concave i-corner and '−' to a convex i-corner
- Remove $(+,-)$ pairs sequentially resulting with signature $- - - \cdots - + \cdots + +$ which is called the i-signature of Y.

We define

$\tilde{f}_i(Y) = Y'$, where Y' is obtained by adding a i-color box at the corner corresponding to the left most $+$ in the i-signature.

$\tilde{e}_i(Y) = Y'$, where Y' is obtained by deleting a i-color box at the corner corresponding to the right most $-$ in the i-signature.

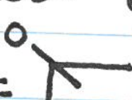
Note that if the i-signature of Y is

$$\underbrace{- - - \cdots - }_{\varepsilon_i(Y)}\underbrace{+ + \cdots + + +}_{\varphi_i(Y)}$$

and $\mathrm{wt}(Y) = \sum_{i=0}^{n-1} (\varphi_i(Y) - \varepsilon_i(Y)) \Lambda_i$

<u>Ex</u>: $B(\Lambda_0)$ crystal for the $U_q(\widehat{sl}(2))$ module $V(\Lambda_0)$.

The highest wt. vector

$v_{\Lambda_0} =$  $= (0, 0, \cdots)$