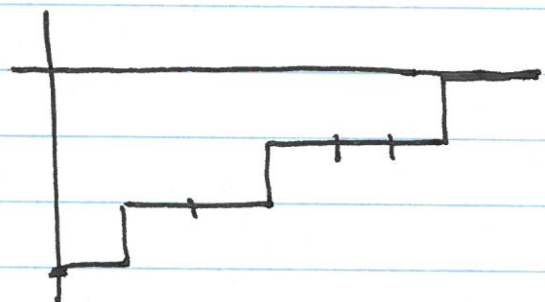
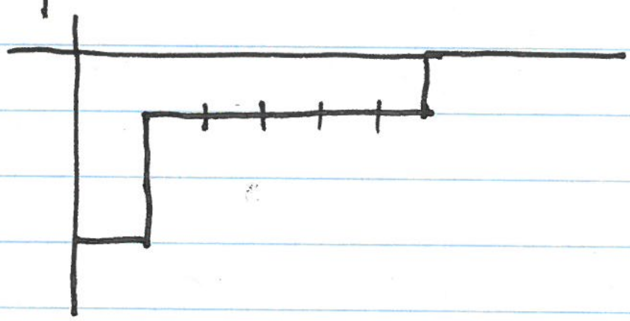
$Y = (y_k)_{k \geq 0}$, $Y' = (y'_k)_{k \geq 0}$ two extended Young diagrams. We say $Y \supseteq Y'$ if ~~$y_k \geq y'_k$~~ ~~$|y_k| \leq |y'_k|$~~ $y_k \leq y'_k$ $\forall k \geq 0$

$Y =$ 

$Y' =$ 

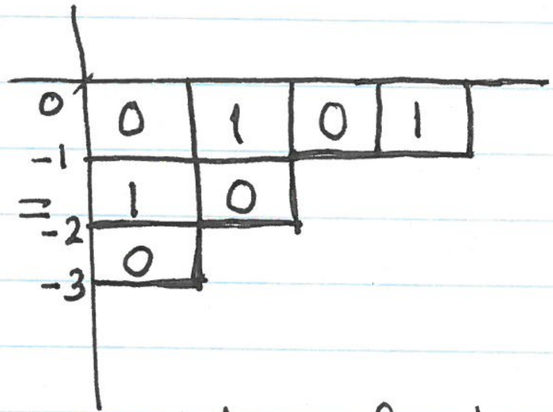
Recall $Y[n] = (y_k + n)_{k \geq 0}$

Defn: $Y = (y_k)_{k \geq 0}$ is n-reduced if ~~as a diagram $Y \supseteq Y[n]$~~

~~as~~ $y_{k+1} - y_k \leq n$ $\forall$ $k \geq 0$

Ex:

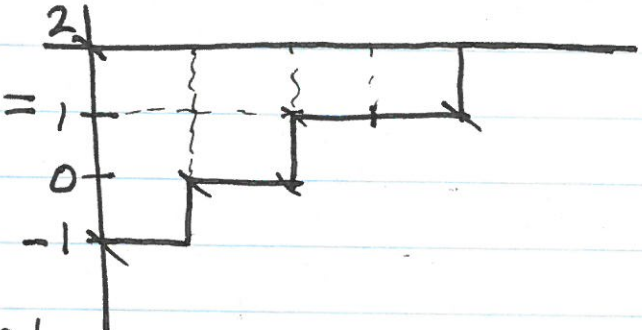
$Y =$



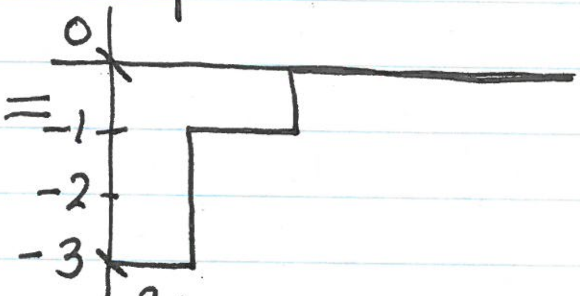
$Y_\infty = 0 \quad , \quad n = 2$

is 2-reduced diagram.

$Y[2] =$



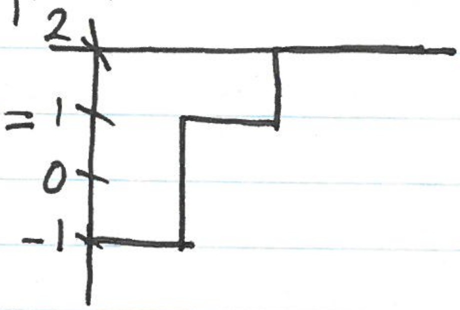
$Y' =$



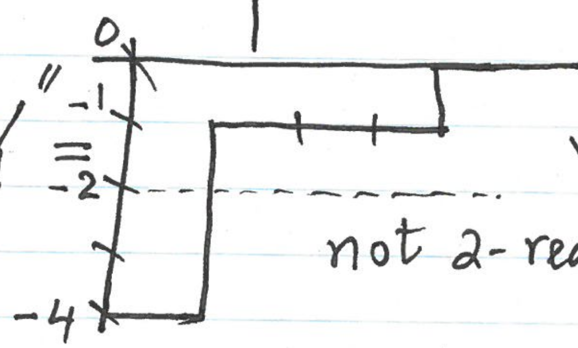
not 2-reduced

~~but as j-reduced form~~

$Y'[2] =$

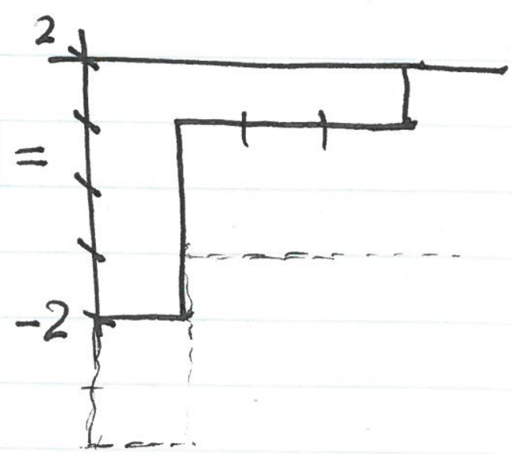


$Y'' =$



not 2-reduced

$Y''[2] =$



$y_1 = -1, \; y_0 = -4$

$y_1 - y_0 = 3 > 2$

$$Y = (y_k)_{k \geq 0}, \quad Y[n] = (y_k + n)_{k \geq 0}$$

We say Y is n-reduced if

$$y_{k+1} - y_k < n, \text{ for } k \geq 0.$$

$$\Lambda_j(h_i) = \delta_{ij}, \quad 1 \leq i, j \leq n.$$

$V(\Lambda_j)$ be the irred. $\mathcal{U}_q(\widehat{sl}(n))$ module with highest weight Λ_j.

$V(\Lambda_j) \in \mathcal{O}^q_{int}$.

Let $(L(\Lambda_j), B(\Lambda_j))$ be its crystal base.

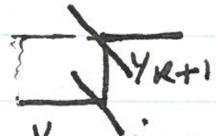
Thm: $B(\Lambda_j) = \{ Y = (y_k)_{k \geq 0} \mid \text{charge } Y = j, \text{ and } Y \text{ is } n\text{-reduced} \}$

For $Y \in B(\Lambda_j)$

$$wt(Y) = \Lambda_j - \sum_{i=0}^{n-1} m_i \alpha_i$$

where m_i = # of i-color boxes in the finite part of Y.

In Y, if $y_k \neq y_{k+1}$ then we have a corner.



i-corner if the corresponding box in the finite diagram is of color i.

is called a convex ~~concave~~ i-corner

is called a concave ~~convex~~ i-corner.

$d \equiv i \pmod{n}$

- Fix $0 \leq i \leq n-1$, $Y \in B(\Lambda_j)$
- Collect the i-corners in Y and order them from right to left (equivalently in decreasing order of their diagonal numbers)
- assign $-$ to a convex i-corner and $+$ to a concave i-corner.