**8th Southeastern Lie Theory workshop on**

**“Algebraic and Combinatorial Representation Theory”**

**NCSU, October 9 – 11, 2015**

**ABSTRACTS**

**Invited Talks:**

**Toshiki Nakashima, Sophia University**

Title: Affine geometric crystal and limit of KR perfect crystals of $A^{(1)}\_n$  
  
Abstract: We shall construct a new geomeric crystal of affine type A  
on certain Schubert cell associated with the translaton $t(\varpi\_k)$, where $t(\varpi\_k)$ is an element of affine Weyl group of type A. Furthermore, we shall see that ultra-discretization of the affine  
geometric crystal is isomorphic to certain limit of KR perfect crystal of affine type A which is constructed by Schilling [et.al](http://et.al/" \t "_blank). This is joint work with K. Misra.

**Masato Okado, Osaka City University**

Title: Tetrahedron equation and generalized quantum groups  
  
Abstract: We construct $2^n$-families of solutions of the Yang-Baxter equation from n-products of certain operators, R and L, satisfying the  
tetrahedron equation. They are identified with the quantum R matrices for the Hopf algebras known as generalized quantum groups. Depending on the number of R's and L's involved in the product, the trace construction interpolates the symmetric tensor representations of $U\_q(A^{(1)}\_{n-1})$ and the anti-symmetric tensor representations of $U\_{-q^{-1}}(A^{(1)}\_{n-1})$, whereas a boundary vector construction interpolates the q-oscillator representation of $U\_q(D^{(2)}\_{n+1})$ and the spin representation of $U\_{-q^{-1}}(D^{(2)}\_{n+1})$. The intermediate cases are associated with an affinization of quantum super algebras. This is a joint work with Atsuo Kuniba and Sergey Sergeev.

**Daniel Orr, Virginia Tech**

Title: Combinatorics of nonsymmetric Macdonald polynomials

Abstract: To any (possibly non-reduced) affine root system one can associate a family of nonsymmetric Macdonald polynomials. These polynomials exhibit many remarkable properties as well as deep connections to geometry and representation theory.

In this talk we shall be concerned with the Ram-Yip formula, which gives a combinatorial expression for the nonsymmetric Macdonald polynomials based on the notion of alcove walks. I will discuss joint work with Mark Shimozono in which we study the behavior of the Ram-Yip formula under various specializations of the parameters in the Macdonald polynomials. Of particular interest are the specializations at t=0 and t=infinity, where it is possible to characterize the surviving terms using the quantum Bruhat graph. I will also discuss a conjectured representation-theoretic interpretation of the nonsymmetric Macdonald polynomials at t=infinity.

**Anne Schilling, UC - Davis**

Lectures I&II: Crystal bases: Stembridge axioms and virtual crystals  
  
Abstract I&II: We will introduce crystal bases for classical Lie algebras. Our approach is through the local characterization of simply-laced crystals given by the Stembridge axioms and the virtual crystal construction for non-simply laced types. This is achieved by showing that both Stembridge crystals and virtual crystals form a tensor category. We will also explain why crystals form such a powerful combinatorial tool for studying representation theory and related questions.  
  
These lectures are based on a book draft in collaboration with Dan Bump.  
  
Lecture III: Crystal approach to affine Schubert calculus  
  
Abstract III: We apply crystal theory to affine Schubert calculus.  
In particular, we define a crystal structure on affine factorizations  
which appear in k-Schur function theory. This yields the Schur expansion of (affine) Stanley symmetric functions and gives crystal operators that intertwine with the Edelmann-Greene insertion (rather than the usual RSK insertion).  
  
This is joint work with Jennifer Morse, see  
International Mathematics Research Notices 2015, doi:10.1093/imrn/rnv194IMRN

**Peter Tingley, Loyola University, Chicago**

Title:  Root multiplicities via quiver varieties and crystals.

Abstract: We discuss a method for estimating root multiplicities for symmetric Kac-Moody algebras. Our work with Baumann and Kamnitzer implies that certain root multiplicities coincide with the number of ``stable'' irreducible components of a quiver variety. We associate to a component its string data, a word in the nodes of the Dynkin diagram. The multiplicity is then the number of words that (i) are valid string data, and (ii) correspond to stable components. Translating these conditions into combinatorics is non-trivial, but in many cases they give simple upper bounds. For instance, in rank 2, the multiplicity is bounded by the number of rational Dyck paths satisfying a simple condition at each corner, which in turn can be approximated by a closed form formula. We conjecture that these upper bounds are very good, and give both computational and heuristic evidence.

**Ben Webster, University of Virginia**

Title: Unexpected gradings in representation theory

Abstract: A lot of interesting algebras and categories that appear in representation theory turn out to have gradings which are not obvious from their original definition.  Examples include the group algebra of the symmetric group over a field of positive characteristic, and category O's for both Lie algebras and Cherednik algebras.  We'll explain how some of these gradings arise and how they enrich our understanding of these representation theories.

**Jerzy Weyman, University of Connecticut**

Title: On the local cohomology of determinantal varieties

Abstract: In this talk I will discuss a joint project with Claudiu Raicu on local cohomology of determinantal varieties. Let *K* be  a filed of characteristic zero. Let *S=K[X\_{i,j}]* be a polynomail ring on the entries of a generic m x n matrix. Denote by *I\_p* the ideal of p x p minors of the generic matrix *X =(X\_{i,j})* over *S*.

We give the complete description of the local cohomology groups *H^i\_{I\_p}(S)*. They turn out to be multiplicity free representations of the group *GL\_m x GL\_n* acting on *S*.

**Contributed Talks:**

**Justin Allman, Duke University**

Title: Grothendieck polynomials as iterated residues

Abstract: Stable Grothendieck polynomials corresponding to partitions represent the classes of Schubert varieties in the K-theory of Grassmannians. It is known that their integer span forms a commutative, cocommutative bialgebra, a result of Buch. We describe the operations of this bialgebra by formulas similar to generating sequences which we call iterated residue formulas. We discuss progress on the application of the iterated residue approach to questions of positivity.

**Iana Anguelova, College of Charleston**

Title: Multilocal bosonization and multilocal fermionization

Abstract: Bosonization, namely the representation of given chiral fields (Fermi or Bose) via bosonic fields, has long been studied both in the physics and the mathematics literature, especially in connection to representation theory. Perhaps the best known instance is the bosonization of the charged free fermions: one of the two directions of an isomorphism often referred to as "the" boson-fermion correspondence. The opposite process, fermionization, refers to the representation of given fields in terms of fermionic fields, as in the case of the representation of the Heisenberg bosonic current as a normal ordered product of the two charged fermions. In this talk we present a new approach, using multi-locality, instead of the usual 1-point locality, to construct new instances of bosonization and fermionization. One is the multilocal bosonization of the $\beta\gamma$ boson ghost system, and generally of the symplectic bosons (and consequently certain classes of lattice vertex algebras). The other is the multilocal fermionization of the charged free fermions, and generally certain other classes of lattice vertex algebras.

**Eric Bucher, Louisiana State University**

Title: Sequences of Quiver Mutations on Surfaces

Abstract: Given a marked surface (S,M) we can add arcs to the surface to create a triangulation, T, of that surface. For each triangulation, T, we can associate a cluster algebra. We will introduce how to construct a cluster algebra and quiver from this surface and hen in the sense of work by Keller we will produce a maximal green sequence for this quiver. Since all finite mutation type cluster algebras can be associated to a surface, with some rare exceptions, this work along with previous work by others seeks to establish a base case in answering the question of whether a given finite mutation type cluster algebra exhibits a maximal green sequence.

**Jason Elsinger, Spring Hill College**

Title: Orbifolds of lattice vertex algebras under an isometry of order two

Abstract: Every isometry of a positive-definite even lattice $Q$ can be lifted to an automorphism of the lattice vertex algebra $V\_Q$. An important problem in vertex algebra theory is to classify the representations of the $\sigma$-invariant subalgebra $V\_Q^\sigma$ of $V\_Q$, known as an orbifold. In the case when $\sigma$ is an isometry of $Q$ of order two, we classify the irreducible modules of the orbifold vertex algebra $V\_Q^\sigma$ and identify them as submodules of twisted or untwisted $V\_Q$ modules. Examples where $Q$ is a root lattice and $\sigma$ is a Dynkin diagram automorphism as well as $Q$ being a direct sum of two copies of a lattice $L$ and $\sigms$ is the cyclic permutation are presented in detail.

**William Hardesty, University of Georgia**

Title: On Support Varieties and the Humphreys Conjecture in type $A$

Abstract: Let $G$ be a reductive algebraic group scheme defined over $\mathbb{F}\_p$ and let $G\_1$ denote the Frobenius kernel of $G$. To each finite-dimensional $G$ module $M$, one can define the support variety $V\_{G\_1}(M)$, which can be regarded as a $G$-stable closed subvariety of the nilpotent cone. A $G$-module is called a tilting module if it has both good and Weyl filtrations. In 1997, it was conjectured by J.E. Humphreys that when $p\geq h$, the support varieties of the indecomposable tilting modules align with the nilpotent orbits given by the Lusztig bijection. We shall verify this conjecture when $G=SL\_n$ and $p > n+1$.

**Thorsten Heidersdorf, Ohio State University**

Title: Ideals in Deligne's category $Rep(O\_{\delta})$ and representations of the Orthosymplectic Supergroup

Abstract: We describe indecomposable objects in Deligne's category $Rep(O\_{\delta})$ and explain how to decompose their tensor products. We then classify thick ideals in $Rep(O\_{\delta})$. As an application we classify the indecomposable summands of tensor powers of the standard representation of the orthosymplectic supergroup up to isomorphism.

**Patricia Hersh, NC State University**

Title: From the weak Bruhat order to the crystal poset  
  
Abstract: We investigate the way in which well-known properties of the weak Bruhat order on a Weyl group can be lifted (or not) to a corresponding crystal graph, viewed as a partially ordered set; the latter projects to the weak order via the so-called key map. First, a crystal theoretic analogue of the statement that any two reduced expressions for the same Coxeter group element are related by Coxeter moves is proven for all lower intervals in a crystal graph. On the other hand, it is shown that no finite set of moves exist, even in type A, for arbitrary crystal graph intervals. Second, it is shown for lower intervals (0,v) that the M ̈obius function is always 0 or ±1, and a precise description is given. For general intervals (u,v), examples exist with arbitrarily large M ̈obius function, again even in type A. Moreover, the order complex for the intervals (0,v) is proven always to be homotopy equivalent to a ball or to a sphere of some dimension, despite often not being shellable. This is joint work with Cristian Lenart.

**Chun-Ju Lai, University of Virginia**

Title: Schur duality, canonical bases, and quantum symmetric pairs of affine type

Abstract:  Recently, generalizing the work of Beilinson, Lusztig, and MacPherson of finite type A, Bao, Kujawa, Li, and Wang constructed the quantum algebras arising from partial flag varieties of finite type B/C, altogether with their canonical bases. They also provided a geometric realization of a Schur-type duality between these algebras and the Hecke algebras of type B acting on a tensor space. These quantum algebras are coideal subalgebras of quantum gl(n), which also form quantum symmetric pairs, with quantum gl(n). The canonical bases arising from quantum symmetric pairs was used by Bao-Wang to formulate Kazhdan-Lusztig theory for BGG category O. The above can be reformulated within the framework of Hecke algebras without geometry.

In this talk I will explain the affinization of above in two (Hecke algebra theoretic and geometric) approaches. This is a joint work with Zhaobing Fan, Yiqiang Li, Li Luo, and Weiqiang Wang.

**David Lax, UNC-Chapel Hill**

Title: Explicit Standard Monomial Basis for Coordinatization of Type A Schubert Varieties

Abstract: A flag manifold for GL(n) can be concretely projectively coordinatized using products of minors as coordinates. These products are nicely indexed by tableaux on a Young diagram. Lakshmibai, Musili, and Seshadri gave a standard monomial basis for the projective coordinates when restricted to a Schubert subvariety. Reiner and Shimozono made this theory more explicit by giving a straightening algorithm for the products of the minors in terms of the right key of a semistandard Young tableau. Since then, Willis introduced scanning tableaux as a more direct way to obtain right keys. We use scanning tableaux to give more-direct proofs of the spanning and the linear independence of the standard monomials. This basis is a weight basis for the dual of a Demazure module for a Borel subgroup of GL(n). As a result, we independently obtain an expression for these Demazure characters as sums of weights over the tableaux used to index the standard monomial bases.

**Jesse Levitt, Louisiana State University**

Title: Connected Hopf algebras as deformations of universal enveloping algebras

Abstract: The classification problem for connected Hopf algebras of finite GK dimension has attracted much attention in recent years. Recently, a classification of all such Hopf algebras of GK dimension 4 was devised in part using Coassociative Lie Algebras to construct deformed universal enveloping algebras. Motivated by this, we invoke a Drinfeld construction to produce non-cocommutative Hopf algebras on completions of universal enveloping algebras. In this joint work with Milen Yakimov, we describe necessary and sufficient conditions for deformed universal enveloping algebras to be connected Hopf Algebras and show how previous classifications follow from the Drinfeld construction.

**Robert Muth, University of Oregon**

Title: Skew Specht modules and cuspidal modules in affine type A

Abstract: A key role in the representation theory of the symmetric group, and cyclotomic Hecke algebras more generally, is played by certain modules called Specht modules, which are labeled by Young diagrams. In the affine setting of the KLR algebra of type A, one may construct a more general class of Specht modules associated to skew diagrams. Cuspidal modules, which form the building blocks of the representation theory of KLR algebras, can be viewed as specific examples of skew Specht modules associated to hook shapes.

**Chad Mangum, Niagara University**

Title: Free Field Representations of Twisted Toroidal Lie Algebras

Abstract: Lie algebra representation theory has been significant in various areas of mathematics and physics for several decades. In this talk, we will discuss one instance of this theory, namely certain representations of twisted (2-)toroidal (Lie) algebras, which we view as universal central extensions of twisted multi-loop algebras. The loop algebra realization generalizes the familiar realization of affine Kac-Moody algebras. To facilitate our study of the representation theory, we will first discuss an alternative presentation of twisted toroidal algebras given via generators and relations. Subsequently, we will discuss a free field representation (which is similar to that of a landmark work by Feingold and Frenkel in the case of affine algebras). This is joint work with Dr. Kailash Misra and Dr. Naihuan Jing.

**Jacob Matherne, Louisiana State University**

Title: Combinatorics of exceptional sequences in type A  
  
Abstract: Exceptional sequences are certain ordered sequences of quiver representations. We introduce a class of objects called strand diagrams and use this model to classify exceptional sequences of representations of a quiver whose underlying graph is a type A\_n Dynkin diagram. We also use variations of this model to classify c-matrices of such quivers, to interpret exceptional sequences as linear extensions of posets, and to give a simple bijection between exceptional sequences and certain chains in the lattice of noncrossing partitions. This is a joint work with Alexander Garver, Kiyoshi Igusa, and Jonah Ostroff.

**Radmila Sazdanovic, NC State University**

Title: Categorification of Chebyshev polynomials

Abstract: We introduce a diagrammatic categorification of the one-variable polynomial ring Z[x] which leads to categorification of some basic special functions such as Chebyshev polynomials.

**Lisa Schneider, Susquehanna University**

Title: Demazure Flags of Demazure Modules  
  
Abstract: In this talk, we study a family of Demazure modules in the affine setting that are also modules of the maximal parabolic subalgebra. Then, we define the Demazure flag of a module for the current algebra and q-multiplicities associated to a Demazure flag. In particular, we are interested in Demazure flags of level m-Demazure modules. For the lie algebra of type A\_1, we consider non-split short exact sequences to define two different recursions of q-multiplicities. In specific cases, we have closed forms for the q multiplicities. Then, we consider generating functions with q-multiplicities as coefficients and observe (when possible) the connections to partial and mock-theta functions as well as Chebyshev polynomials.

**Paul Sobaje, University of Georgia**

Title: Lifting representations for algebraic groups

Abstract: We will look at the problem of determining when a module for a Frobenius kernel of an algebraic group $G$ lifts to $G$.

**Xin Tang, Fayetteville State University**

Title: Automorphisms of Multiparameter Quantized Weyl Algebras

Abstract: In this talk, we give complete description of the automorphism group of a class of multiparameter quantized Weyl algebras and their central extensions. A few applications will be discussed as well.

**Ryan Vinroot, College of William and Mary**

Title: Kostka multiplicity one for multipartitions  
  
Abstract: If $[\lambda(j)]$ is a multipartition of the positive integer $n$ (a sequence of partitions with total size $n$), and $\mu$ is a partition of $n$, we study the number $K\_{[\lambda(j)]\mu}$ of sequences of semistandard Young tableaux of shape $[\lambda(j)]$ and total weight $\mu$. We show that the numbers $K\_{[\lambda(j)] \mu}$ occur naturally as the multiplicities in certain permutation representations of wreath products. The main result is a set of conditions on $[\lambda(j)]$ and $\mu$ which are equivalent to $K\_{[\lambda(j)] \mu} = 1$, generalizing a theorem of Berenshte\u{\i}n and Zelevinski\u{\i}. We also show that the questions of whether $K\_{[\lambda(j)] \mu} > 0$ or $K\_{[\lambda(j)] \mu} = 1$ can be answered in polynomial time, expanding on a result of Narayanan. Finally, we give an application to multiplicities in the degenerate Gel'fand-Graev representations of the finite general linear group, and we show that the problem of determining whether a given irreducible representation of the finite general linear group appears with nonzero multiplicity in a given degenerate Gel'fand-Graev representation, with their partition parameters as input, is $NP$-complete.

**Evan Wilson, Ursinus College**

Title: Completely pointed modules of U\_q(sl\_n)

Abstract: Completely pointed modules are weight modules whose weight spaces are one dimensional. In this talk, we give a classification of completely pointed modules of the quantum group for U\_q(sl\_n) which was recently obtained by the author with Vyacheslav Futorny and Jonas Hartwig.